Proof that the single-variable linear-regression predictor derived using the general matrix-based multiple regression algorithm gives the same results as the original Pyret implementation.

Given: a set of inputs $\{x, \ldots\}$ and their corresponding outputs $\{y, \ldots\}$.

Let

$$X = \begin{bmatrix} 1 & x \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}, Y = \begin{bmatrix} y \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

Using the multiple-regression algorithm, we get

$$B = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \left(X^T X\right)^{-1} X^T Y. \qquad (1)$$

and the predictor function is $y = \alpha + \beta x$.

We have

$$X^T = \begin{bmatrix} 1 & \cdot & \cdot & \cdot \\ x & \cdot & \cdot & \cdot \end{bmatrix}$$

$$\therefore X^T X = \begin{bmatrix} 1 & \cdot & \cdot & \cdot \\ x & \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} 1 & x \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} = \begin{bmatrix} n & \Sigma x \\ \Sigma x & \Sigma x^2 \end{bmatrix}$$

We then have

$$\det\ X^T X = n \Sigma x^2 - (\Sigma x)^2 = \Delta \text{ (say)}$$

$$\text{and} \qquad \text{cof}\ X^T X = \begin{bmatrix} \Sigma x^2 & -\Sigma x \\ -\Sigma x & n \end{bmatrix}$$

The adjoint of a matrix is the transpose of its cofactor matrix. So

$$\text{adj}\ X^T X = \left(\text{cof}\ X^T X\right)^T$$

But cof $X^T X$ is diagonally symmetric, so its transpose is itself. So

$$\text{adj}\ X^T X = \text{cof}\ X^T X$$

The inverse of a matrix is its adjoint divided by its determinant. So

$$\left(X^T X\right)^{-1} = \frac{\text{adj}\ X^T X}{\Delta} = \left(\frac{1}{\Delta}\right) \begin{bmatrix} \Sigma x^2 & -\Sigma x \\ -\Sigma x & n \end{bmatrix}$$

Putting all this in (1), we have

$$\begin{aligned} B &= \left(\frac{1}{\Delta}\right) \begin{bmatrix} \Sigma x^2 & -\Sigma x \\ -\Sigma x & n \end{bmatrix} \begin{bmatrix} 1 & \cdot & \cdot & \cdot \\ x & \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} y \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \\ &= \left(\frac{1}{\Delta}\right) \begin{bmatrix} \Sigma x^2 & -\Sigma x \\ -\Sigma x & n \end{bmatrix} \begin{bmatrix} \Sigma y \\ \Sigma xy \end{bmatrix} \\ &= \left(\frac{1}{\Delta}\right) \begin{bmatrix} \Sigma x^2 \Sigma y - \Sigma x \Sigma xy \\ -\Sigma x \Sigma y + n \Sigma xy \end{bmatrix} \end{aligned}$$

$$\therefore \alpha = \frac{\Sigma x^2 \Sigma y - \Sigma x \Sigma xy}{n \Sigma x^2 - (\Sigma x)^2}$$

$$\text{and } \beta = \frac{n \Sigma xy - \Sigma x \Sigma y}{n \Sigma x^2 - (\Sigma x)^2} \qquad (2)$$

Back to the original Pyret implementation. There we have

$$\begin{aligned} \beta &= \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\Sigma x^2 - \frac{(\Sigma x)^2}{n}} \\ &= \frac{n \Sigma xy - \Sigma x \Sigma y}{n \Sigma x^2 - (\Sigma x)^2} \end{aligned}$$

$$\begin{aligned} \text{and} \qquad \alpha &= \bar{y} - \beta \bar{x} \\ &= \left(\frac{\Sigma y}{n}\right) - \left(\frac{n \Sigma xy - \Sigma x \Sigma y}{n \Sigma x^2 - (\Sigma x)^2}\right)\left(\frac{\Sigma x}{n}\right) \\ &= \left(\frac{\Sigma y}{n}\right) - \left(\frac{n \Sigma x \Sigma xy - (\Sigma x)^2 \Sigma y}{n\left(n \Sigma x^2 - (\Sigma x)^2\right)}\right) \\ &= \frac{\Sigma y\left(n \Sigma x^2 - (\Sigma x)^2\right) - n \Sigma x \Sigma xy + (\Sigma x)^2 \Sigma y}{n\left(n \Sigma x^2 - (\Sigma x)^2\right)} \\ &= \frac{n \Sigma x^2 \Sigma y - (\Sigma x)^2 \Sigma y - n \Sigma x \Sigma xy + (\Sigma x)^2 \Sigma y}{n\left(n \Sigma x^2 - (\Sigma x)^2\right)} \\ &= \frac{n \Sigma x^2 \Sigma y - n \Sigma x \Sigma xy}{n\left(n \Sigma x^2 - (\Sigma x)^2\right)} \\ &= \frac{\Sigma x^2 \Sigma y - \Sigma x \Sigma xy}{n \Sigma x^2 - (\Sigma x)^2} \end{aligned}$$

But these match exactly the values for α, β in (2). QED.